## Precalc Warm Up # 10-2

1. Find the exact value using a sum formula:

   $\cos(285^{\circ})$

2. Solve: $3\tan^2 x + 4\tan x = 4$

   (use radians)

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In Exercises 29 and 30, use the power-reducing formulas to write each expression in terms of the first power of the cosine.

29. (a) $\cos^4 x$ $\longrightarrow$ $(\cos^2 x)^2$

    (b) $\sin^2 x \cos^4 x$

$$= \left(\frac{1+\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)$$

$$= \frac{1 + 2\cos 2x + \cos^2(2x)}{4}$$

$$= \frac{1}{4} + \frac{\cos 2x}{2} + \frac{1}{4}\left(\frac{1+\cos 4x}{2}\right)$$

$$= \frac{2}{2}\cdot\frac{1}{4} + \frac{\cos 2x}{2} + \frac{1}{8} + \frac{\cos 4x}{8}$$

$$= \boxed{\frac{3}{8} + \frac{\cos 2x}{2} + \frac{\cos 4x}{8}}$$

**75.** $\cos 2x = \cos x$

**77.** $\sin 4x + 2\sin 2x = 0$

**79** $(\sin 2x + \cos 2x)^2 = 1$

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$\sin 2(2x) + 2\sin 2x = 0$ ($u$ above $2x$)

$2\sin 2x\cos 2x + 2\sin 2x = 0$

$2\sin 2x(\cos 2x + 1) = 0$

$\sin 2x = 0$ $\qquad$ $\cos 2x = -1$

$2x = \sin^{-1}(0)$ $\qquad$ $2x = \cos^{-1}(-1)$

$2x = \pi n$ $\qquad$ $2x = \pi + 2\pi n$

$x = \frac{\pi}{2}n$ $\qquad$ $x = \frac{\pi}{2} + \pi n$

$x = 0, \frac{\pi}{2}, \frac{3\pi}{2}, \pi$ $\qquad$ $x = \frac{\pi}{2}, \frac{3\pi}{2}$

79) $(\sin 2x + \cos 2x)^2 = 1$

$\underline{\sin^2 2x} + 2\sin 2x\cos 2x + \underline{\cos^2 2x} = 1$

Since $\sin^2 2x + \cos^2 2x = 1$

subtract 1 from both sides

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#3 $\dfrac{\sin^2\alpha - \cos^2\alpha}{\sin^2\alpha - \sin\alpha\cos\alpha} \longleftarrow$ factor difference of squares

9) $1 - 4\sin^2 x\cos^2 x$

$1 - (2\sin x\cos x)^2$

13) $\sin^5 x\cos^2 x =$

$\sin x(\sin^4 x)\cos^2 x =$

$\sin x(\sin^2 x)^2\cos^2 x =$

$\sin x(1-\cos^2 x)^2\cos^2 x =$

$\sin x(1 - 2\cos^2 x + \cos^4 x)\cos^2 x =$

$\sin x(\cos^2 x - 2\cos^4 x + \cos^6 x = \quad . \; . \; . \; .$

19) $\sqrt{\dfrac{\phantom{1+\sin\theta}}{1+\sin\theta}} \cdot \dfrac{1-\sin\theta}{1-\sin\theta}$

21) $\cos 3x = 4\cos^3 x - 3\cos x$

$\cos(2x+x) =$

$\cos 2x \cos x - \sin 2x \sin x =$

$(2\cos^2 x - 1)(\cos x) - 2\sin x \cos x \sin x =$

$2\cos^3 x - \cos x - 2\sin^2 x \cos x =$

$2\cos^3 x - \cos x - 2(1-\cos^2 x)\cos x =$

$2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x =$

$4\cos^3 x - 3\cos x =$

Take the three power reducing formulas and replace u with $\frac{u}{2}$. Square root both sides. These are the Half Angle Formulas.

$$\sin^2 u = \frac{1-\cos 2u}{2} \qquad \cos^2 u = \frac{1+\cos 2u}{2} \qquad \tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

$$\left(\sin \frac{u}{2}\right)^2 = \frac{1-\cos 2\left(\frac{u}{2}\right)}{2}$$

$$\sqrt{\left(\sin \frac{u}{2}\right)^2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

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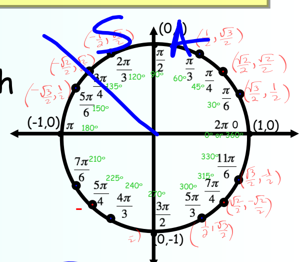
Half Angle Formulas:

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}} \qquad \tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

The sign depends upon the quadrant in which $\frac{u}{2}$ lies. For example, find cos $105^{\circ}$ exactly




$$105^{\circ} = \frac{210^{\circ}}{2}$$

$$\cos\frac{210}{2} = \pm\sqrt{\frac{1+\cos 210}{2}} = \pm\sqrt{\frac{1+-\frac{\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{2-\sqrt{3}}{4}} = \boxed{-\frac{\sqrt{2-\sqrt{3}}}{2}}.$$

and since cos 105 is negative, the answer is negative.

Check with your calculator!

How else could we have found cos $105^{\circ}$ exactly?

$$\cos(60^{\circ} + 45^{\circ})$$

Find the general solution to $2 - \sin^2 x = 2\cos^2 \frac{x}{2}$

power reduce

$$2 - \sin^2 x = 2\left(\frac{1+\cos x}{2}\right)$$

$$2 - \sin^2 x = 1 + \cos x$$

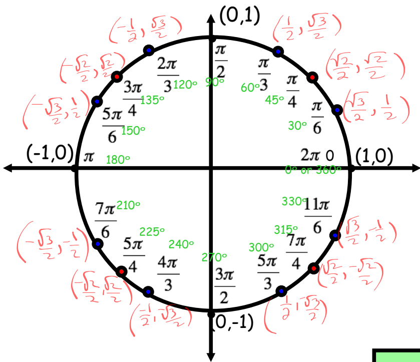
$$2 - (1 - \cos^2 x) = 1 + \cos x$$

$$2 - 1 + \cos^2 x = 1 + \cos x$$

$$\cos^2 x - \cos x = 0$$

$$\cos x(\cos x - 1) = 0$$

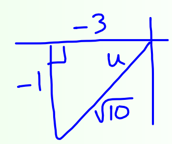
$$\cos x = 0 \text{ or } \cos x = 1$$




$$\boxed{x = \pi/2 + \pi n \text{ and } x = 2\pi n}$$

Find the exact value of $\sin\left(\frac{u}{2}\right)$ when $\cot u = 3$, $\pi < u < \frac{3\pi}{2}$

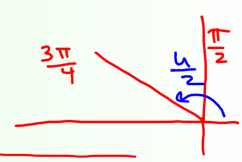
$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$




$$= \sqrt{\frac{1-\left(-\frac{3}{\sqrt{10}}\right)}{\frac{2}{1}}} \qquad \sqrt{\frac{1}{2}\left(\frac{\sqrt{10}}{\sqrt{10}} + \frac{3}{\sqrt{10}}\right)}$$

$$\pi < u < \frac{3\pi}{2}$$

$$= \sqrt{\frac{\sqrt{10}+3}{2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}}$$




$$= \sqrt{\frac{10+3\sqrt{10}}{20}} \quad \text{OK...}$$

Best: $\dfrac{\sqrt{50+15\sqrt{10}}}{10}$

The sign of your answer depends on where $\frac{u}{2}$ is. How do we know where it is? Quad II, sine is +

To finish up our yellow sheets, we need the Product to Sum Formulas and the Sum to Product formulas

Product to Sum:

sin u sin v =(1/2)[cos (u - v) - cos (u + v)]
cos u cos v =(1/2)[cos (u - v) + cos (u + v)]
sin u cos v =(1/2)[sin (u + v) + sin (u - v)]
cos u sin v =(1/2)[sin ( u + v) - sin ( u - v)]

Verify the first one!

$$\sin u \sin v = \frac{1}{2}\left[\cos u \cos v + \sin u \sin v - (\cos u \cos v - \sin u \sin v)\right]$$

$$= \frac{1}{2}\left[2 \sin u \sin v\right]$$

$$\sin u \sin v = \sin u \sin v$$

Rewrite the product as a sum and simplify:

product to sum

sin u sin v =(1/2)[cos (u - v) - cos (u + v)]
cos u cos v =(1/2)[cos (u - v) + cos (u + v)]
sin u cos v =(1/2)[sin (u + v) + sin (u - v)]
cos u sin v =(1/2)[sin ( u + v) - sin ( u - v)]

$$4 \sin \frac{\pi}{3} \cos \frac{5\pi}{6}$$

$$4\left(\frac{1}{2}\right)\left[\sin\left(\frac{2}{2}\cdot\frac{\pi}{3} + \frac{5\pi}{6}\right) + \sin\left(\frac{2}{2}\cdot\frac{\pi}{3} - \frac{5\pi}{6}\right)\right]$$

$$2\left(\sin \frac{7\pi}{6} + \sin\left(\frac{-3\pi}{6}\right)\right)$$

$$2\left(-\frac{1}{2}\right) + 2(-1)$$

$$\boxed{-3}$$

Writing sums as products can be particularly useful. Why??

Sum to product:

These can be easily proven using the product to sum formulas.
Prove the first one on your own.

Verify the identity using the product to sum identities :

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$= \cancel{2}\left(\cancel{\tfrac{1}{2}}\right)\left[\sin\left(\frac{x+y}{2} + \frac{x-y}{2}\right) + \sin\left(\frac{x+y}{2} - \frac{x-y}{2}\right)\right]$$

$$= \sin\frac{2x}{2} + \sin\frac{2y}{2}$$

$$= \sin x + \sin y$$

product to sum

$\sin u \sin v = (1/2)[\cos(u - v) - \cos(u + v)]$

$\cos u \cos v = (1/2)[\cos(u - v) + \cos(u + v)]$

$\sin u \cos v = (1/2)[\sin(u + v) + \sin(u - v)]$

$\cos u \sin v = (1/2)[\sin(u + v) - \sin(u - v)]$

Find the solutions to sin 5x + sin 3x = 0 [0, 2π)

change to a product

$$2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)=0$$

$$2\sin(4x)\cos(x)=0$$

sin 4x = 0 or cos x = 0

$4x = 0 + \pi n$
$x = \pi n/4$

$x = \pi/2$ , $3\pi/2$

$\boxed{x = 0, \pi/2, \pi/4, 3\pi/2, \pi/2, 3\pi/4, \pi, 5\pi/4, 7\pi/4}$

HW: p. 440 # 13 - 25 odd,

31 - 49 odd, 81, 83

Group Test Thursday
Individual Test Friday